VARIOUS FORMS OF DROP FRACTIONATION IN SHOCK WAVES AND THEIR SPECIAL CHARACTERISTICS

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The principal forms of drop fractionation behind shock waves are considered; empirical relationships are established for the most important parameters of the fractionation process.

Damage suffered by the surfaces of turbine blades or supersonic aircraft in gas – liquid media, the transient and explosive combustion of fuel in gas – liquid systems, and the relaxational interaction of pressure waves with two-phase systems are frequently determined by the character of the drop-fractionation process. The most important parameters in the fractionation or break-up process are the critical conditions corresponding to the development of various forms of fractionation, the fractionation time, and the spectrum of the particles created after the damage has occurred. Fractionation processes have been studied by a number of authors over the last few years, and many theoretical and experimental papers have been written on this subject. Analysis shows that the process of drop fractionation cannot be given any adequate theoretical description at the present time. The only feature capable of being established theoretically is the quantitative value of the rupture criterion $We^* \approx 5$–8 [1–5]. No useful information regarding the times and varieties of the fractionation process has yet been obtained as a result of theoretical investigations, although a number of attempts have been made [4, 5].

An idea as to the dynamics of drop breakdown and the parameters chiefly influencing these may be obtained experimentally. It was shown in [6, 7] that for small supercritical Weber numbers the breakdown of a drop took place by way of the blowing out of a thin film of liquid in the center ("parachute" breakdown). For large Weber numbers $We > 500$ the breakdown of the drop involves the separation of the liquid boundary layer [8–10]. The question as to the breakdown of drops in the intermediate range of Weber numbers $We^* < We < 500$ has never been resolved. An analysis of transient processes may indicate the reasons for the appearance of any particular form of drop breakdown. Another important aspect is that of proving the universality of the criteria and experimental relationships employed over a wide range of variation of the parameters of a two-phase flow. Existing data mainly relate to drop fractionation at ambient pressures of 10^{-3} to 30 atm, the shock wave Mach numbers M ranging from 1.1 to ~5 and the Reynolds numbers lying in the range $10 < Re < 10^4$. The range of drop sizes for which most of the experiments were conducted was extremely narrow: $d \approx 0.2$–2 mm. In the present investigation we sought to extend this range of drop sizes.

Results of the Experiments. The characteristics of drop breakdown in shock waves were studied in a shock tube of rectangular cross section. The parameters of the waves (velocity and intensity) were determined by means of piezoelectric pressure sensors. The drops were photographed on a stationary film using a pulsed light source connected at a particular instant of time. As subject for study we used water and kerosene drops from 80 μ to 1 mm in size. The error in measuring the velocity of the shock waves was ~10% and the error in measuring the drop size ~6%.

Figure 1 shows a series of photographs of a water drop 500 μ in size breaking up in a gas flow behind a wave with a Mach number of M = 1.09; the Weber and Reynolds numbers are, respectively, We = 12 and Re = 1500. The photographs of Fig. 1, Ia–e were taken at the instants 0, 320, 450, 560, and 900 μsec after the wave had encountered the drop. In these pictures we see the principal characteristics of the "parachute" breakdown of the drops. During the first 500 μsec the drop deforms into a disk, then a thin liquid

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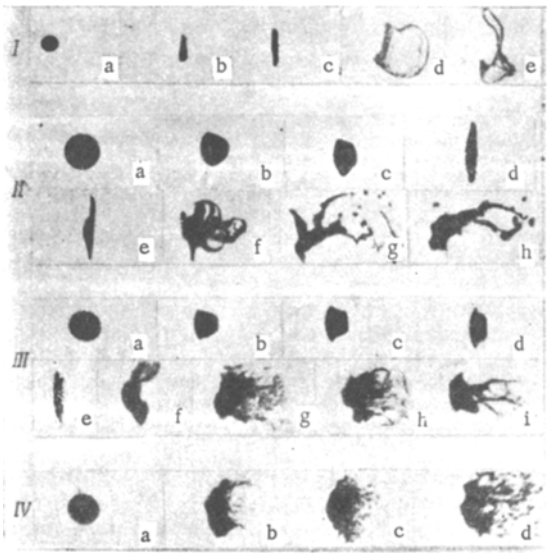

Fig. 1. Photographs of drops breaking up.

film is blown out from the disk. Owing to the accelerated motion of the film, acceleration waves are generated upon its surface. In Fig. 1 the waves of instability have the form of alternating light and dark concentric circles. The development of instability leads to the decomposition of the film and to the formation of a large number of small drops. For some while a liquid torus with a continuously increasing cross section exists in the gas flow. After the break-up of the torus the fractionation process may be regarded as completed.

Let us take the dimensionless time $t^* = tt_0^{-1}$ as defining parameter. Here $t_0 = d_0\rho_f^{0.5}(\rho u^2)^{-0.5}$. At the instant of time $t^* \approx 1.5$ the greatest transverse deformation of the drop $d^* = dd_0^{-1} \approx 2$ is achieved. At the same time a thin film is blown out. Rupture of the film occurs at $t^* \approx 2.3$. The liquid torus exists in the gas flow for a period $t^* \approx 2.3$-3.5. At a time $t^* \approx 3.5$-4 the breakdown of the drop ends and the drop is able to move through a distance $x \approx 20\ d_0$.

Figure 2 shows the size distribution of the number of drops obtained in the break-up process. The vertical axis gives the relative quantity $n^* = n_i n^{-1}$ and the horizontal axis gives $d^* = d_i d_0^{-1}$ in which d_i is the size of the drops belonging to the i-th group. For measuring the spectrum the microdrops were divided into dimensional groups with a step of 20 μ. It was assumed that the drops inside each group had a constant diameter of d_i. The dimensions of the original drops $d_0 = 0.5$ mm (curve 1) and $d_0 = 0.9$ mm (curve 2). We see from Fig. 2 that two groups may be distinguished in the spectrum: large and small. The small drops have a size of $0.1\ d_0$ and are obtained after break-up of the thin film; the large drops $(0.2\text{-}0.3)d_0$ in size are formed when the torus breaks up. Some 70% of the mass of the original drop goes into the large drops. The "parachute" type of drop fractionation occurs in the range of Weber numbers $We^* < We < 15$ for kerosene and water drops with a dimension of $d_0 = 80$-$1000\ \mu$.

Increasing the Weber number to values exceeding $We_i^* \approx 15$ leads to a change in the character of the fractionation process. Figure 1, II illustrates the rupture of a water drop 1 mm in size. The Weber number is We = 33 and the Reynolds number Re = 4000. Figure 1, IIa-h indicate the state of the drop at the instants of time 0, 200, 330, 630, 670, 1100, 1800, and 1950 μsec after the wave has met the drop. For 1000 μsec deformation of the original particle takes place, then several thin films are formed in the drop. After the rupture of the films the drop acquires an indefenite form and breaks up into filaments which transform into fine drops. As the drop undergoes deformation, the greatest cross section of $d^* \approx 2.3$ is attained at the instant of time $t^* \approx 1.5$. The fractionation is completed at the instant $t^* \approx 4$-5, and the drop is able to move through a distance $x \approx 25\ d_0$. In the fractionated spectrum drops of size $0.1\ d_0$ tend to predominate. The foregoing phenomena occur over a narrow range of Weber numbers. A slight increase in the rate of gas flow leads to the appearance of a new type of fractionation process. Figure 1, III illustrates the break-up of a kerosene drop $d_0 = 1.1$ mm in size in a gas flow with a Weber number We = 57 and a Reynolds number Re = 3050. The state of the drop is illustrated for the instants of time 0, 130, 215, 280, 360, 710, 860, 970, and 1250 μsec. A characteristic feature of the drop deformation process is the formation of a sharp edge along the periphery of the drop. A similar effect (through poorly defined) occurs for a water drop at times 200-300 μsec, as indicated in Fig. 1, IIc and d. The surface tension of water is three times greater than that of kerosene. The radius of curvature of the sharp edge is therefore smaller for water. It is highly likely that the form of the sharp edge on the equator of the drop is a consequence of the formation of a boundary layer in the liquid owing to the existence of frictional forces at the phase interface. The growth of the boundary layer and the deformation of the drop take place at the same time. After reaching the maximum stage of deformation a liquid film starts peeling from the surface of the drop; the film breaks up owing to its instability on acceleration in the gas flow. The remaining unbroken part of the drop assumes a random shape and decomposes after $t \approx 2000\ \mu$sec, i.e., $t^* \approx 4$-5.

An increase in the relative velocity of the gas and the drops leads to an intensification of the foregoing processes. Figure 1, IV shows the rupture of a water drop $d_0 = 1$ mm in size for a Weber number We = 90 and a Reynolds number Re = 3600. The photographs of Fig. 1, IVa-d were taken at the moments of time 0, 270, 470, and 1160 μsec. In view of the large relative velocity the rupture of the film takes place

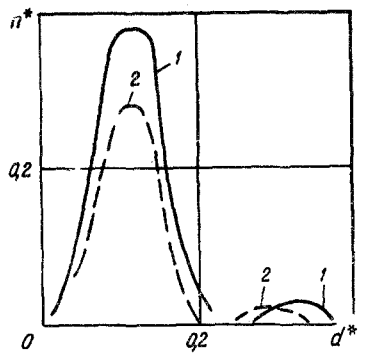

Fig. 2

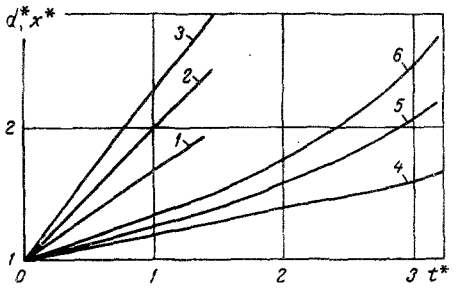

Fig. 3

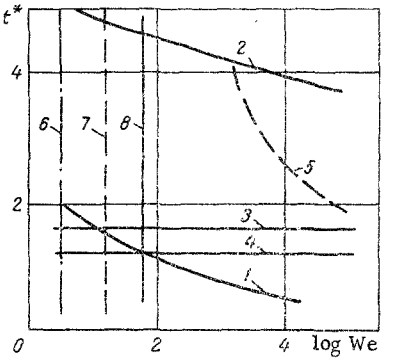

Fig. 4

Fig. 2. Size distribution of the drops.

Fig. 3. Deformation and displacement of the drops breaking up.

Fig. 4. Times corresponding to deformation, boundary-layer formation, and instability-wave growth as functions of the Weber number.

close to the surface of the drop, while the fractionation process appears as a separation of the microdrops from the original drop. The maximum deformation of the drops $d^* = 2.9$ occurs at time $t^* \approx 1.5$. Complete breakdown of the drop occurs at $t^* \approx 5$ at a distance of $x \approx 30\,d_0$. After the detachment of the thin liquid film, drops $0.1\,d_0$ in size are created. The greater part of the drop breaks up later as a result of the undulating distortion of the "windward" surface, $0.15\,d_0$ particles being formed. The break-up of the liquid drops with the separation of the thin film occurs for Weber numbers of $We_2^* \approx 40$-60. Reducing the diameter of the drops leads to a reduction in the Weber number at which the break-up of the drop is accompanied by peeling of the liquid film. The results of the experiments and the data presented in [8-10] enable us to derive simple analytical relationships for the time changes in the cross section and movement of the drops, and also to determine the instants corresponding to the initial and final break-up of the drops in the Weber number range $We^* < We < 10^4$. The inducation time of the fractioning process may be calculated from the equation $\tau_i = 1.5\,d_0\,\rho_f^{0.5}(\rho u^2)^{-0.5}(\log We)^{-0.25}$. The complete break-up of the drops occurs in a time $\tau_B = 5\,d_0\,\rho_f^{0.5}(\rho u^2)^{-0.5}(\log We)^{-0.1}$. The time dependence of the drop deformation and the displacement of the drops during break-up are illustrated in Fig. 3. Here the vertical axis represents the degree of deformation of the drop d^* and the displacement of the drop $x^* = xd_0^{-1}$, while the horizontal axis represents the dimensionless time t^*. Curve 1 shows the degree of deformation of the drop, and curve 4 its displacement during "parachute" break-up. Curves 2 and 5 correspond to the deformation and displacement of the drop in random break-up, curves 3 and 6 to the break-up of the drop involving the separation of the surface layer of liquid.

The change taking place in the transverse dimension of the drop with time may be expressed in the form $d^* = 1 + (1 - We^*We^{-1})t^*$. The relationship in question is valid from Weber numbers of We^* to $We = 2 \cdot 10^4$, Reynolds numbers of $Re = 400$ to $Re \approx 10^4$, drop sizes of $d_0 = 80\,\mu$ to $d_0 = 3$ mm, shock-wave Mach numbers of $M = 1.05$ to $M = 4$, and for water, kerosene, alcohol, and heptane drops. The equation holds true within the time range $0 < t^* < 1.5$.

The dimensionless displacement of the drops may be written in the following way: $x = C_D 0.375\,t^*$. Comparing this equation with the experimental data, we find the resistance coefficient of the disintegrating drops C_D. When the drops breaks up on the "parachute" principle the resistance coefficient $C_D \approx 1.6$; for chaotic disintegration it is 1.9, and for break-up with peeling of the surface layer of liquid 2.2. A value of $C_D \approx 3$ was given in [9, 10] for Reynolds numbers of $Re > 10^4$. The generalized relationship for calculating the resistance coefficients will be $C_D \approx 0.26\,Re^{0.25}$.

Discussion of Results. In the range of variation of Weber numbers $5 < We < 300$ there are at least three forms of drop break-up characterized by different rates of drop deformation, different mechanisms of drop motion under the influence of the gas flow, and different spectra of the microdrops formed by the disintegration of the original drop. Hence apart from the critical Weber number We^* reflecting the resistance of the drop to the action of the gas flow there should clearly be two further critical conditions which determine the transition from one form of drop break-up to another.

The reasons for the existence of several types of break-up are to be linked with the complex combined action of the gas flow on the drop. Let us analyze the principal phenomena accompanying the action of the gas flow on the drop. Owing to the flow of gas around the drop a pressure distribution is created around the latter, with a minimum on the equator. The pressure distribution is the principal reason for the deformation of the drop. The flow of gas over the surface of the drop sets the surface layer of liquid in motion and a boundary layer is formed in the liquid. Finally the drop acquires an acceleration in the flow of gas; on the windward side of the drop this is directed from the gas into the liquid, which promotes the development of Taylor instability at the phase interface. Only due allowance for the combined development of all these phenomena enables us to understand every feature of the drop break-up process.

Each of the phenomena indicates possesses characteristic time and scale parameters. We shall characterize the drop deformation process by the degree of deformation d* and the deformation time τ. We define the formation of the boundary layer of liquid by its thickness δ and time of formation τ_s. We characterize the instability of the phase interface by the wavelength λ and the time of development of the instability τ_λ. If for a certain mode of flow around the drop it so happens that one of the phenomena takes place more rapidly than all the others, then the break-up of the drop will be due to the fastest process. Thus it may be found that the depth of the boundary layer δ or the wavelength of the instability λ exceed the dimensions of the drop d_0, and for such dimensions of the boundary layer, or for such wavelengths, neither the formation of the boundary layer nor the instability of the interface will play any part in the breakdown of the drops.

The deformation of a drop in a gas flow was studied in [1-4, 7]. The characteristic time required to attain the critical deformation is $t^* \approx 1.4$, while the maximum degree of deformation $d^* \approx 2.8$. The time required to form the boundary layer in the liquid $\tau_s t_0^{-1} = \tau_s^* \approx 1.2$ is known from [8, 9]. The depth of the boundary layer at the drop is defined as $\delta^* = 2\delta d_0^{-1} = 3.44\,(\rho_f \mu_f^2)^{0.25}\,(\rho\mu^2)^{-0.25}\,\mathrm{Re}^{0.5}$. The size of the drop $0.5\,d_0$ and the thickness of the boundary layer δ are equal for Weber numbers of $\mathrm{We} \approx 50\ \mathrm{Lp}^{-1}$. For drops of diameter $d_0 = 50\text{-}1000\,\mu$ this corresponds to Weber numbers $\mathrm{We} < 5$.

The condition for the separation of the boundary layer of liquid from the drop may be found by equating the force of the dynamic gas-pressure head (which causes the formation and peeling of the boundary layer of liquid) to the force of surface tension in a layer of thickness δ which prevents the separation of the boundary layer of liquid $0.5\,\beta\rho u^2 = 2\psi\delta^{-1}$, where $\beta \approx 0.3$ is the coefficient of wind resistance. Solving the resultant equation we come to the relationship $\mathrm{We} = 1.5\,(\rho_f\mu^2)^{0.25}\,(\rho\mu_f^2)^{-0.25}\,\mathrm{Re}^{0.5}$. The separation of the boundary layer of liquid from kerosene drops at $p_0 = 1$ atm takes place on satisfying the condition $\mathrm{We} > 0.8\,\mathrm{Re}^{0.5}$, in agreement with the conclusions of [12]. We express the Weber number in the form $\mathrm{We} = 0.5\,\mathrm{Re}^2\,\rho_f\mu^2\,(\rho\mu_f^2)^{-1}\,\mathrm{Lp}^{-1}$ and obtain the condition for the separation of the surface layer of liquid $\mathrm{We} \geq 2.5\,\mathrm{Lp}^{0.33}$. Experimental data confirm the validity of this relationship.

The development of the Taylor instability on the surface of an accelerated drop was analyzed in [8, 14, 15]. The minimum instability wavelength is given by $\lambda_m = 2\pi\psi^{0.5}\,(a\rho_f)^{-0.5}$. The acceleration of the drop may be found by using the experimental resistance coefficient $C_D \approx 1.5$. We then obtain $\lambda_m^* = \lambda_m d_0^{-1} = (17\,\mathrm{We}^{-1})^{0.5}$. The length of the unstable wave on the surface of the drop is greater than the diameter of the drop for $\mathrm{We} \leq 17$. As indicated in [15] the time of development of the instability for large Bond numbers is $\tau_\lambda^* = \tau_\lambda t_0^{-1} \approx 22\,\mathrm{Bo}^{-0.25}$. The Bond number is uniquely expressed in terms of the Weber number and the characteristic development time of the acceleration wave $\tau_\lambda^* \approx 22\,\mathrm{We}^{-0.25}$.

A comparison of the scale parameters for the three foregoing processes indicates that in the range of Weber number $\mathrm{We} < \mathrm{We}^*$ break-up is impossible, since $\lambda > d_0$, $\delta \approx d_0$, $d^* = 1$. In the range $\mathrm{We}^* < \mathrm{We} < \mathrm{We}_1^* = 15$ the condition $d^* > 1$ is satisfied, and this defines the possibility of the deformation process. In the range of Weber numbers $\mathrm{We}_1^* < \mathrm{We} < 2.5\,\mathrm{Lp}^{0.33}$ the relationships $\lambda < d_0$, $d^* > 1$ is satisfied, i.e., the processes of deformation and instability development are allowed. The relationships $\lambda < d_0$, $\delta^* < 0.5$, $d^* > 1$ are valid when $\mathrm{We} > 2.5\,\mathrm{Lp}^{0.33}$. The satisfaction of the foregoing inequalities defines the possibility of the simultaneous occurrence of all three processes. A comparison between the characteristic development times of deformation, boundary-layer formation, and instability development are illustrated in Fig. 4. The vertical axis indicates the dimensionless time t*, the horizontal axis the logarithm of the Weber number. Curves 1 and 2 describe the change in the time corresponding to the beginning and end of the drop break-up with increasing Weber number; curves 6, $\mathrm{We} = 5$; 7, $\mathrm{We}_1^* \approx 15$; 8, $\mathrm{We}_2^* \approx 2.5\,\mathrm{Lp}^{0.33}$ (for kerosene drops with $d_0 = 1$ mm) divide the range of Weber numbers into three parts. Straight lines 3 and 4 show the time required to reach the critical stage of drop deformation τ^* and the time required to form the boundary layer of liquid τ_s^*. Curve 5 corresponds to the time of development of the acceleration wave τ_λ^*. In the

range of Weber numbers $5 < We < 15$ the time to the onset of drop break-up is no shorter than the time required to reach critical deformation. The reason for the break-up of the drop is its continuous deformation.

In the Weber number range $15 < We \leq 2.5\ Lp^{0.33}$ break-up of the drop becomes possible as a result of the deformation and instability of the windward surface of the drop. The closeness of the characteristic deformation time τ^* to the induction time τ_i^* shows that the onset of break-up is associated with the attachment of the critical stage of deformation. At the final stage of break-up the instability of the windward side of the drop is the important feature.

In the range of Weber numbers $We > 2.5\ Lp^{0.33}$ the induction time is close to the time of formation of the boundary layer in the liquid. The time required to reach the critical stage of deformation and the time for the development of the acceleration wave are shorter than the time required for complete drop break-up. The break-up of the drop starts on account of the detachment of the surface layer of liquid; however, the mass of the drop then diminishes by no more than 20% over a time $t^* \approx 2.5$-3 [8]. The main reason for the break-up of the drop is deformation and the development of instability on the windward side of the drop. The maximum rate of break-up of the drop in the time $t^* \approx 2$-3 [10, 16] corresponds to the instant at which the amplitude of the acceleration waves becomes comparable with the minimum dimensions of the flattened drop. In Fig. 1 this instant of time is masked by the cloud of microdrops formed in the breakdown of the peeled surface layer of liquid. The characteristic time for the development of instability in the range of Weber numbers $2 \cdot 10^3 < We < 2 \cdot 10^4$ is $1.8 < \tau_\lambda^* < 3.5$ [10, 16, 17]. With increasing Weber number the characteristic time for the development of the acceleration waves diminishes. For Weber numbers $We > 2 \cdot 10^5$ the breakdown of the drop will probably be mainly caused by the instability of the windward surface of the drop. Under these conditions the characteristic times for the development of deformation and the formation of the boundary liquid layer are longer than the time of development of the acceleration waves [14].

Thus the existence of different forms of drop break-up over a wide range of variation of the Weber numbers is determined by the combined action of deformation, boundary-layer formation in the liquid, and the development of instability on the windward surface of the drop. The characteristics of each form of break-up depend on the dynamics of one such process or of the whole set. A more detailed understanding of the mechanism of drop break-up may be obtained by analyzing the combined action of a large number of processes accompanying the break-up of the drop (for example, allowing for the circulation of the liquid in the drop, eddy motion behind the drop, etc).

NOTATION

u	is the gas velocity;
ρ	is the gas density;
μ	is the gas viscosity;
ψ	is the surface tension of the liquid;
d_0	is the initial drop diameter;
$We = \rho u^2 d_0 (2\psi)^{-1}$	is the Weber number;
$Re = \rho u d_0 \mu^{-1}$	is the Reynolds number;
M	is the Mach number;
ρ_f	is the density of liquid;
x	is the distance;
n_i	is the number of drops of size d_i;
C_D	is the resistance (drag) coefficient;
t	is the time;
τ_s	is the time of formation of the boundary layer;
λ	is the wavelength;
τ	is the time of wave development;
μ_f	is the viscosity of the liquid;
$Lp = d_0 \rho_f \psi / \mu_f^2$	is the Laplace number;
p_0	is the gas pressure;
a	is the acceleration of the drop;
$Bo = 0.25\ \rho_f a d_0^2 \psi^{-1}$	is the Bond number.

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